$$|\Psi\rangle$$

Physics Formula Sheet

Natural Constants

Speed of light	$c = 2.998 \times 10^{8}$ m/s	Planck constant	$h = 6.626 \times 10^{-34}$ J·s
Reduced Planck $\hbar = h/2\pi$	$\hbar = 1.055 \times 10^{-34}$ J·s	Boltzmann constant	$k_B = 1.381 \times 10^{-23}$ J/K
Avogadro number	$N_A = 6.022 \times 10^{23}$ mol^{-1}	Gas constant $R = N_A k_B$	$R = 8.314$ J/(mol·K)
Elementary charge	$e = 1.602 \times 10^{-19}$ C	Electron mass	$m_e = 9.109 \times 10^{-31}$ kg
Proton mass	$m_p = 1.6726 \times 10^{-27}$ kg	Neutron mass	$m_n = 1.6749 \times 10^{-27}$ kg
Gravitational constant	$G = 6.674 \times 10^{-11}$ N·m^2/kg^2	Standard gravity	$g = 9.807$ m/s^2
Vacuum permittivity	$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m	Vacuum permeability	$\mu_0 = 1.257 \times 10^{-6}$ H/m
Coulomb constant $k_e = 1/4\pi\varepsilon_0$	$k_e = 8.988 \times 10^{9}$ N·m^2/C^2	Stefan-Boltzmann	$\sigma = 5.671 \times 10^{-8}$ W/(m^{2}K^4)
Bohr radius	$a_0 = 5.292 \times 10^{-11}$ m	Rydberg energy	$E_R = 13.606$ eV
Fine-structure constant	$\alpha = e^2/(4\pi\varepsilon_0\hbar c) \approx 1/137.036$	Wien constant	$b = 2.898 \times 10^{-3}$ m·K
Atomic mass unit	$u = 1.661 \times 10^{-27}$ kg	Proton–electron mass ratio	$m_p/m_e = 1836.15$

1 Mechanics

1.1 Kinematics

$$\mathbf{v} = \dot{\mathbf{r}}, \quad \mathbf{a} = \dot{\mathbf{v}}$$
$$v = v_0 + at, \quad x = x_0 + v_0 t + \tfrac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a\,\Delta x$$

r position [m], **v** velocity [m/s], **a** acceleration [m/s^2]

1.2 Dynamics & Conservation

$$\mathbf{F} = m\mathbf{a} = \dot{\mathbf{p}}, \quad \mathbf{p} = m\mathbf{v}$$
$$\mathbf{J} = \Delta\mathbf{p} = \int \mathbf{F}\,dt \quad \text{(impulse)}$$
$$\mathbf{F}_{\text{grav}} = -\frac{GMm}{r^2}\hat{r}, \quad U_{\text{grav}} = -\frac{GMm}{r}$$

p momentum [kg·m/s], **J** impulse [N·s], G gravitational constant

Friction

$$f_s \le \mu_s N, \quad f_k = \mu_k N$$

μ_s, μ_k static/kinetic friction coefficients, N normal force

1.3 Work, Energy & Power

$$W = \int \mathbf{F} \cdot d\mathbf{s}$$
$$E_k = \tfrac{1}{2}mv^2, \quad U_{\text{spring}} = \tfrac{1}{2}kx^2$$
$$E_k + U = \text{const} \quad \text{(conservative forces)}$$
$$P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$$

W work [J], k spring constant [N/m], P power [W]

1.4 Rotational Mechanics

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = I\boldsymbol{\alpha}$$
$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}, \quad \boldsymbol{\tau} = \dot{\mathbf{L}}$$
$$E_{\text{rot}} = \tfrac{1}{2}I\omega^2$$
$$I = \int r^2\,dm, \quad I = I_{\text{cm}} + md^2 \text{ (parallel axis)}$$

$\boldsymbol{\tau}$ torque [N·m], I moment of inertia [kg·m^2], $\boldsymbol{\omega}$ angular velocity [rad/s], **L** angular momentum [kg·m^2/s]

Common Moments of Inertia

Solid sphere	$\frac{2}{5}mR^2$	Hollow sphere	$\frac{2}{3}mR^2$
Solid cylinder	$\frac{1}{2}mR^2$	Thin ring	mR^2
Rod (centre)	$\frac{1}{12}mL^2$	Rod (end)	$\frac{1}{3}mL^2$

1.5 Oscillations

$$\ddot{x} + \omega_0^2 x = 0 \;\Rightarrow\; x(t) = A\cos(\omega_0 t + \varphi)$$
$$\omega_0 = \sqrt{k/m}\ \text{(spring)}, \quad \omega_0 = \sqrt{g/L}\ \text{(pendulum)}$$
$$T = 2\pi/\omega_0, \quad f = \omega_0/2\pi$$

ω_0 natural frequency [rad/s], A amplitude, φ initial phase, T period [s]

Damped & Forced Oscillation

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t$$
$$\omega' = \sqrt{\omega_0^2 - \gamma^2}, \quad x_{\text{damp}} = Ae^{-\gamma t}\cos(\omega' t + \varphi)$$
$$A_{\text{res}} = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\gamma^2\omega^2}}$$

γ damping coefficient, ω' damped frequency, A_{res} forced amplitude

2 Electrodynamics

2.1 Electrostatics

$$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{r} \quad \text{(Coulomb)}$$
$$\mathbf{E} = -\nabla V, \quad V = \frac{q}{4\pi\varepsilon_0 r}$$
$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{Q_{\text{enc}}}{\varepsilon_0} \;\Leftrightarrow\; \nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0} \quad \text{(Gauss)}$$
$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}, \quad u_E = \tfrac{1}{2}\varepsilon_0 E^2$$

E electric field [V/m], V potential [V], ρ charge density [C/m^3], u_E energy density [J/m^3]

Capacitance

$$C = Q/V, \quad U = Q^2/2C = \tfrac{1}{2}CV^2, \quad C_{\text{pp}} = \varepsilon_0 A/d$$

C capacitance [F], A plate area [m^2], d separation [m]

Dielectrics

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P} = \varepsilon_r\varepsilon_0\mathbf{E}, \quad \nabla\cdot\mathbf{D} = \rho_{\text{free}}$$

D displacement field [C/m^2], **P** polarisation, ε_r relative permittivity

2.2 Magnetostatics

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{d\mathbf{l}\times\hat{r}}{r^2} \quad \text{(Biot-Savart)}$$
$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \;\Leftrightarrow\; \nabla\times\mathbf{B} = \mu_0\mathbf{J} \quad \text{(Ampère)}$$
$$\mathbf{F} = q\mathbf{v}\times\mathbf{B} = I\mathbf{L}\times\mathbf{B} \quad \text{(Lorentz / Laplace)}$$

B magnetic flux density [T], I current [A]

Useful Field Results

Long wire	$B = \mu_0 I/2\pi r$
Solenoid (n turns/m)	$B = \mu_0 n I$
Magnetic dipole moment	$\boldsymbol{\mu} = I\mathbf{A}$, $\boldsymbol{\tau} = \boldsymbol{\mu}\times\mathbf{B}$

Inductance & Magnetic Energy

$$\mathcal{E} = -L\dot{I}, \quad U = \tfrac{1}{2}LI^2, \quad u_B = B^2/2\mu_0$$

L inductance [H], $\mathcal{E}$ EMF [V], u_B energy density [J/m^3]

2.3 Maxwell's Equations (SI)

$$\nabla\cdot\mathbf{E} = \rho/\varepsilon_0$$
$$\nabla\cdot\mathbf{B} = 0$$
$$\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t$$
$$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\varepsilon_0\,\partial\mathbf{E}/\partial t$$

J current density [A/m^2]

2.4 Circuits

$$V = IR, \quad P = IV = I^2R = V^2/R$$
$$R = \rho l/A, \quad \rho = \rho_0[1 + \alpha(T - T_0)]$$
$$Z_R = R, \quad Z_C = 1/i\omega C, \quad Z_L = i\omega L$$
$$\tau_{RC} = RC, \quad \tau_{RL} = L/R, \quad \omega_0 = 1/\sqrt{LC}$$

ρ resistivity [Ω·m], l length, A cross-section, Z impedance [Ω], τ time constant

2.5 Electromagnetic Waves

$$c = 1/\sqrt{\mu_0\varepsilon_0}, \quad E_0 = cB_0$$
$$\mathbf{S} = \mu_0^{-1}\mathbf{E}\times\mathbf{B} \quad \text{(Poynting vector)}$$
$$\langle S\rangle = \varepsilon_0 c E_0^2/2 = cB_0^2/2\mu_0 \quad \text{(intensity)}$$
$$P_{\text{rad}} = \langle S\rangle/c\ \text{(absorbing)}, \quad P_{\text{rad}} = 2\langle S\rangle/c\ \text{(reflecting)}$$

S Poynting vector [W/m^2], E_0, B_0 amplitudes

3 Thermodynamics

3.1 Laws of Thermodynamics

$$dU = \delta Q - \delta W, \quad \delta W = p\,dV \quad \text{(1st)}$$
$$dS \geq \delta Q/T\ \text{(2nd, Clausius)}, \quad S \to 0 \text{ as } T \to 0\ \text{(3rd)}$$

U internal energy [J], Q heat [J], W work [J], S entropy [J/K], T temperature [K]

3.2 Ideal Gas

$$pV = nRT = Nk_BT$$
$$U = \tfrac{f}{2}Nk_BT \quad (f \text{ degrees of freedom})$$
$$C_V = \tfrac{f}{2}R, \quad C_p = C_V + R, \quad \gamma = C_p/C_V$$

n moles, N molecules, f: 3 (monatomic), 5 (diatomic, rigid), γ adiabatic index

Thermodynamic Processes

Isothermal	$T = \text{const}$	$W = nRT\ln(V_2/V_1)$
Isochoric	$V = \text{const}$	$W = 0,\ \Delta U = Q$
Isobaric	$p = \text{const}$	$W = p\Delta V$
Adiabatic	$Q = 0$	$pV^\gamma = \text{const},\ TV^{\gamma-1} = \text{const}$

Carnot Efficiency

$$\eta_{\text{Carnot}} = 1 - T_C/T_H, \quad \text{COP}_{\text{refrigerator}} = T_C/(T_H - T_C)$$

T_C cold, T_H hot reservoir temperature [K]

3.3 Thermodynamic Potentials

$$\begin{aligned} H &= U + pV \quad \text{(enthalpy)} \\ F &= U - TS \quad \text{(Helmholtz free energy)} \\ G &= H - TS = U - TS + pV \quad \text{(Gibbs free energy)} \\ dG &= -S\,dT + V\,dp + \mu\,dN \end{aligned}$$

μ chemical potential [J/mol], H enthalpy [J]; equilibrium: $dG = 0$ at const T, p

3.4 Statistical Mechanics

$$\begin{aligned} S &= k_B \ln\Omega \quad \text{(Boltzmann entropy)} \\ Z &= \sum_i e^{-E_i/k_BT}, \quad \beta = 1/k_BT \\ \langle E\rangle &= -\partial\ln Z/\partial\beta, \quad F = -k_BT\ln Z \end{aligned}$$

Ω number of microstates, Z canonical partition function

Maxwell–Boltzmann Speed Distribution

$$f(v) = 4\pi n\left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2 e^{-mv^2/2k_BT}$$

$$v_{\text{mp}} = \sqrt{2k_BT/m}, \quad \langle v\rangle = \sqrt{8k_BT/\pi m}, \quad v_{\text{rms}} = \sqrt{3k_BT/m}$$

Heat Transfer

$$\begin{aligned} \dot{Q}_{\text{cond}} &= -\kappa A\,\nabla T \text{ (conduction)} \\ \dot{Q}_{\text{rad}} &= \sigma AT^4 \text{ (black body)}, \quad \dot{Q}_{\text{conv}} = hA\Delta T \end{aligned}$$

κ thermal conductivity [W/(m·K)], h convective coefficient [W/(m^{2}K)]

4 Quantum Mechanics

4.1 Wave–Particle Duality

$$\begin{aligned} E &= hf = \hbar\omega, \quad p = h/\lambda = \hbar k \\ \lambda_{\text{dB}} &= h/p \quad \text{(de Broglie wavelength)} \end{aligned}$$

f frequency [Hz], λ wavelength [m], $k = 2\pi/\lambda$ wave number [m^{-1}]

4.2 Schrödinger Equation

$$\begin{aligned} i\hbar\frac{\partial\Psi}{\partial t} &= \hat{H}\Psi \quad \text{(time-dependent)} \\ \hat{H}\psi &= E\psi \quad \text{(time-independent)} \\ \hat{H} &= -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) \end{aligned}$$

Ψ wave function, $\hat{H}$ Hamiltonian operator, $V(\mathbf{r})$ potential energy

Normalisation & Expectation Values

$$\int_{-\infty}^{\infty} |\Psi|^2\,dx = 1, \quad \langle\hat{A}\rangle = \int \Psi^*\hat{A}\Psi\,dx$$

Common 1D Solutions

Particle in box ($0 < x < L$)	$E_n = n^2\pi^2\hbar^2/2mL^2,\ n = 1, 2, \ldots$
Harmonic oscillator	$E_n = \hbar\omega_0(n + \frac{1}{2}),\ n = 0, 1, 2, \ldots$

4.3 Uncertainty Principle

$$\Delta x\,\Delta p \geq \hbar/2, \quad \Delta E\,\Delta t \geq \hbar/2$$

Δ standard deviation of observable

4.4 Operators

$$\begin{aligned} \hat{x} = x, \quad \hat{p} = -i\hbar\nabla, \quad \hat{E} = i\hbar\partial_t \\ [\hat{x}, \hat{p}] = i\hbar, \quad [\hat{L}_i, \hat{L}_j] = i\hbar\varepsilon_{ijk}\hat{L}_k \end{aligned}$$

4.5 Angular Momentum

$$\begin{aligned} \hat{L}^2 Y_l^m = \hbar^2 l(l+1) Y_l^m, \quad \hat{L}_z Y_l^m = \hbar m Y_l^m \\ l = 0, 1, 2, \ldots; \quad m = -l, \ldots, +l \\ \hat{S}^2|s, m_s\rangle = \hbar^2 s(s+1)|s, m_s\rangle, \quad \hat{S}_z|s, m_s\rangle = \hbar m_s|s, m_s\rangle \end{aligned}$$

Y_l^m spherical harmonics, l orbital q.n., m magnetic q.n., s spin q.n.

4.6 Hydrogen Atom

$$\begin{aligned} E_n &= -\frac{E_R}{n^2} = -\frac{13.606\text{ eV}}{n^2}, \quad n = 1, 2, 3, \ldots \\ a_0 &= \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = 5.292 \times 10^{-11}\text{ m} \end{aligned}$$

n principal q.n., $E_R = 13.606$ eV Rydberg energy, a_0 Bohr radius

Spectral Series & Selection Rules

$$\begin{aligned} \frac{1}{\lambda} = R_\infty\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), \quad R_\infty = 1.097 \times 10^7\text{ m}^{-1} \\ \Delta l = \pm 1, \quad \Delta m = 0, \pm 1, \quad \Delta s = 0 \text{ (E1 rules)} \end{aligned}$$

5 Special Relativity

5.1 Lorentz Transformation (S' moves at v along x)

$$t' = \gamma(t - vx/c^2), \quad x' = \gamma(x - vt)$$
$$\gamma = 1/\sqrt{1-\beta^2}, \quad \beta = v/c$$
$$\Delta t = \gamma\,\Delta t_0 \text{ (time dilation)}, \quad L = L_0/\gamma \text{ (length contraction)}$$

γ Lorentz factor, Δt_0 proper time, L_0 proper length

Relativistic Velocity Addition

$$u' = \frac{u - v}{1 - uv/c^2}$$

5.2 Relativistic Energy & Momentum

$$E = \gamma mc^2, \quad E_0 = mc^2, \quad E_k = (\gamma - 1)mc^2$$
$$\mathbf{p} = \gamma m\mathbf{v}$$
$$E^2 = (pc)^2 + (mc^2)^2 \quad \text{(dispersion relation)}$$

E_0 rest energy, E_k kinetic energy, m invariant rest mass

5.3 Spacetime Interval & Doppler

$$s^2 = -c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 \text{ (invariant interval)}$$
$$f_{\text{obs}} = f_{\text{src}}\sqrt{\frac{1 \mp \beta}{1 \pm \beta}} \quad (-/+\text{: receding/approaching})$$

6 Fluid Dynamics

6.1 Fundamental Equations

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0 \quad \text{(continuity)}$$
$$A_1 v_1 = A_2 v_2 \quad \text{(incompressible flow)}$$
$$p + \tfrac{1}{2}\rho v^2 + \rho g z = \text{const} \quad \text{(Bernoulli)}$$

ρ density [kg/m^3], $\mathbf{v}$ velocity field [m/s], p pressure [Pa], z height [m]

6.2 Viscous Flow

$$\tau_{\text{shear}} = \eta\, dv/dy \quad \text{(Newton's viscosity law)}$$
$$\text{Re} = \rho v L/\eta \quad \text{(Reynolds number)}$$
$$Q = \pi R^4 \Delta p / 8\eta L \quad \text{(Hagen-Poiseuille)}$$

η dynamic viscosity [Pa·s], Re $\lesssim 2300$ laminar, Re $\gtrsim 4000$ turbulent, Q flow rate [m^3/s]

Navier–Stokes (incompressible, $\nabla\cdot\mathbf{v} = 0$)

$$\rho(\partial_t\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v}) = -\nabla p + \eta\nabla^2\mathbf{v} + \mathbf{f}$$

$\mathbf{f}$ body force per unit volume

Drag, Buoyancy & Surface Tension

$$F_{\text{Stokes}} = 6\pi\eta r v \text{ (sphere)}, \quad F_b = \rho_f V_{\text{sub}} g$$
$$\Delta p = 2\gamma/R \text{ (bubble)}, \quad \Delta p = 4\gamma/R \text{ (soap film)}$$

r sphere radius, γ surface tension [N/m], R radius of curvature

7 Optics

7.1 Geometric Optics

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{(Snell's law)}$$
$$\theta_B = \arctan(n_2/n_1) \quad \text{(Brewster's angle)}$$
$$\sin\theta_c = n_2/n_1 \quad \text{(critical angle, TIR for } n_1 > n_2\text{)}$$

n refractive index, θ angle from surface normal

Thin Lens & Mirror Equations

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}, \quad m = -d_i/d_o$$
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{(lensmaker's equation)}$$

f focal length, d_o object distance, d_i image distance ($+$: real, $-$: virtual), m magnification

7.2 Wave Optics

Double-Slit Interference (Young)

$$d\sin\theta = m\lambda \text{ (max)}, \quad d\sin\theta = (m + \tfrac{1}{2})\lambda \text{ (min)}$$
$$\Delta y = \lambda L/d \quad \text{(fringe spacing)}$$

d slit separation, λ wavelength, $m \in \mathbb{Z}$, L screen distance

Single-Slit Diffraction

$$a\sin\theta = m\lambda \text{ (minima)}, \quad I(\theta) = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2, \quad \alpha = \frac{\pi a \sin\theta}{\lambda}$$

a slit width, first minimum at $\sin\theta = \lambda/a$

Diffraction Grating & Resolving Power

$$d\sin\theta = m\lambda, \quad \mathcal{R} = \lambda/\Delta\lambda = mN$$

d grating spacing, N total number of slits, $\mathcal{R}$ resolving power

Thin Film Interference

$$2nt = m\lambda \text{ (constructive, no net phase shift)}$$
$$2nt = (m + \tfrac{1}{2})\lambda \text{ (destructive)}$$

n film index, t thickness; reflection from denser medium adds $\lambda/2$ phase shift

Photons

$$c = f\lambda, \quad E_{\text{photon}} = hf = hc/\lambda, \quad p_{\text{photon}} = h/\lambda = E/c$$

8 Particle Physics

8.1 Radioactive Decay

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{-t/\tau}$$
$$\lambda = \ln 2/t_{1/2}, \quad \tau = 1/\lambda, \quad A = \lambda N$$

λ decay constant [s^{-1}], τ mean lifetime [s], $t_{1/2}$ half-life [s], A activity [Bq]

Q-Value of a Reaction

$$Q = (m_i - m_f)c^2 = K_f - K_i$$

$Q > 0$: exothermic/exoergic; $Q < 0$: endothermic, requires threshold energy

8.2 Standard Model Particles

Quarks (×3 colours)	$u, d\ (Q = +\frac{2}{3}, -\frac{1}{3}), c, s, t, b$
Leptons	$e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau$ (+ antiparticles)
Gauge bosons	γ (EM), $W^\pm, Z^0$ (weak), g gluons (strong, 8)
Higgs	$H^0, m_H \approx 125$ GeV/c^2

9 Solid State Physics

9.1 Crystal Structure & Diffraction

$$2d \sin\theta = m\lambda \quad \text{(Bragg's law)}$$
$$\mathbf{G}\cdot\mathbf{R} = 2\pi n \quad \text{(reciprocal lattice condition)}$$

d plane spacing, θ Bragg angle, **G** reciprocal lattice vector, **R** lattice vector, $n \in \mathbb{Z}$

9.2 Free Electron Model (Drude)

$$\sigma = ne^2\tau/m_e, \quad \mathbf{J} = \sigma\mathbf{E}$$
$$\mu_e = e\tau/m_e \quad \text{(mobility)}$$

σ conductivity [Ω^{-1}m^{-1}], τ scattering time, n carrier density [m^{-3}], μ_e mobility [m^2/(V·s)]

Fermi Energy (3D, $T = 0$)

$$E_F = \frac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}, \quad g(E) = \frac{3n}{2E_F}\left(\frac{E}{E_F}\right)^{1/2}$$

E_F Fermi energy, $g(E)$ density of states [J^{-1}m^{-3}]

Hall Effect

$$R_H = V_H/(IB) = -1/(ne) \quad (R_H > 0 \text{ for holes})$$

V_H Hall voltage, sign determines carrier type

9.3 Semiconductors

$$n_i^2 = np = N_C N_V e^{-E_g/k_B T}$$
$$I = I_0(e^{eV/k_B T} - 1) \quad \text{(Shockley diode equation)}$$

n_i intrinsic carrier density, E_g band gap, N_C, N_V effective DOS at band edges

9.4 Phonons & Lattice Specific Heat

$$C_V^{\text{Debye}} \propto T^3 \text{ (low } T), \quad C_V \to 3Nk_B \text{ (Dulong-Petit, high } T)$$
$$E = \hbar\omega_k \quad \text{(phonon energy)}$$

θ_D Debye temperature; Dulong-Petit: classical limit $3R \approx 24.9$ J/(mol·K)

9.5 Superconductivity

$$R = 0 \text{ for } T < T_c \quad \text{(zero resistance)}$$
$$\mathbf{B} = 0 \text{ inside superconductor (Meissner effect)}$$

T_c critical temperature; above T_c or critical field B_c: superconductivity breaks down

10 Plasma Physics

10.1 Plasma Parameters

$$\lambda_D = \sqrt{\varepsilon_0 k_B T/ne^2} \quad \text{(Debye shielding length)}$$
$$\omega_p = \sqrt{ne^2/\varepsilon_0 m_e} \quad \text{(electron plasma frequency)}$$
$$\omega_{c,e} = eB/m_e \quad \text{(electron cyclotron frequency)}$$
$$r_L = mv_\perp/eB \quad \text{(Larmor / gyroradius)}$$

n plasma density [m^{-3}], λ_D Debye length; plasma condition: $n\lambda_D^3 \gg 1$

10.2 Plasma Waves & Cutoffs

$$\omega^2 = \omega_p^2 + k^2c^2 \quad \text{(EM wave in unmagnetised plasma)}$$
$$n_{\text{refr}} = \sqrt{1 - \omega_p^2/\omega^2} \quad \text{(refractive index)}$$

Wave evanescent (n_{refr} imaginary) when $\omega < \omega_p$; cutoff at $\omega = \omega_p$

11 Astrophysics

11.1 Orbital Mechanics & Kepler

$$T^2 = \frac{4\pi^2}{GM}a^3 \quad \text{(Kepler's 3rd law)}$$
$$v_{\text{orb}} = \sqrt{GM/r}, \quad v_{\text{esc}} = \sqrt{2GM/R}$$
$$E_{\text{orbit}} = -GMm/2a \quad \text{(total mechanical energy)}$$

T period, a semi-major axis, r orbital radius, R body radius, v_{esc} escape speed

11.2 Stellar Physics

$$L = 4\pi R_\star^2 \sigma T_{\text{eff}}^4 \quad \text{(luminosity)}$$
$$\lambda_{\max} T = b = 2.898 \times 10^{-3} \text{ m·K} \quad \text{(Wien's law)}$$
$$F = L/4\pi d^2 \quad \text{(flux from source at distance } d)$$
$$m_1 - m_2 = -2.5\log_{10}(F_1/F_2) \quad \text{(Pogson's law)}$$

L luminosity [W], $R_\star$ stellar radius, T_{eff} effective temperature, F flux [W/m^2], m apparent magnitude

Distance Modulus

$$\mu = m - M = 5\log_{10}(d/10\,\text{pc})$$

M absolute magnitude, d distance in parsecs; 1 pc = 3.086×10^{16} m

11.3 Black Holes & General Relativity

$$ds^2 = -f\,c^2 dt^2 + f^{-1} dr^2 + r^2 d\Omega^2$$
$$f = 1 - r_s/r, \quad r_s = 2GM/c^2 \text{ (event horizon)}$$
$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad \text{(Einstein field equations)}$$

r_s Schwarzschild radius, Λ cosmological constant, $T_{\mu\nu}$ stress-energy tensor, $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$

Gravitational Redshift & Time Dilation

$$z_{\text{grav}} \approx \Delta\Phi/c^2 = GM/Rc^2, \quad \Delta t_{\text{far}} = \Delta t_{\text{near}}/\sqrt{1 - r_s/r}$$

11.4 Cosmology

$$v = H_0 d \quad \text{(Hubble–Lemaître law)}, \quad H_0 \approx 70\,\text{km/s/Mpc}$$
$$H = \dot{a}/a \quad \text{(Hubble parameter)}, \quad z = \lambda_{\text{obs}}/\lambda_{\text{em}} - 1$$

H_0 Hubble constant, $a(t)$ cosmic scale factor, z cosmological redshift