Global Quantum Mechanics Challenge

Qualification Round

Edition of 2026

A: de Broglie Wavelength (5 Points)

An electron (mass $m_e = 9.109 \times 10^{-31}$ kg, charge $e = 1.602 \times 10^{-19}$ C) is accelerated from rest through a potential difference of $V = 150$ V in a vacuum.

1. Calculate the kinetic energy gained by the electron in joules and in electron-volts.
2. Determine the speed v of the electron after acceleration (treat the motion as non-relativistic; verify that $v \ll c$).
3. Calculate the de Broglie wavelength $\lambda = h/p$ of the accelerated electron, where $h = 6.626 \times 10^{-34}$ J s and $p = m_e v$.
4. A crystal lattice has a typical atomic spacing of $d \approx 2 \times 10^{-10}$ m. Compare λ to d and explain what this implies about the behaviour of electrons in a crystal diffraction experiment.

B: Particle in an Infinite Square Well (5 Points)

A particle of mass m is confined in a one-dimensional *infinite square well* of width L, defined by the potential

$$V(x) = \begin{cases} 0 & 0 \le x \le L, \\ \infty & \text{otherwise.} \end{cases}$$

Inside the well the stationary-state wavefunctions take the form $\psi_n(x) = A \sin(n\pi x/L)$, with $n = 1, 2, 3, \ldots$

1. Determine the normalisation constant A by requiring $\int_0^L |\psi_n(x)|^2 \, dx = 1$.
2. Show that the energy eigenvalues are $E_n = \dfrac{n^2\pi^2\hbar^2}{2mL^2}$.
3. For an electron ($m_e = 9.109 \times 10^{-31}$ kg) in a well of width $L = 1.0$ nm, calculate the ground-state energy E_1 in eV. Use $\hbar = 1.055 \times 10^{-34}$ J s.
4. Calculate the wavelength of the photon emitted when the electron transitions from the state $n = 3$ to the state $n = 1$.

C: Heisenberg Uncertainty Principle (5 Points)

The Heisenberg uncertainty principle states that for *any* quantum state,

$$\Delta x\,\Delta p \;\geq\; \frac{\hbar}{2}, \qquad \Delta E\,\Delta t \;\geq\; \frac{\hbar}{2}, \qquad \hbar = 1.055\times10^{-34}\ \text{J s}.$$

1. An electron's position is known to within $\Delta x = 0.10$ nm. Calculate the minimum uncertainty in its momentum, $(\Delta p)_{\text{min}}$.
2. Estimate the minimum kinetic energy $E_{\text{min}} = \dfrac{(\Delta p)^2_{\text{min}}}{2m_e}$ and express your answer in eV.
3. A proton ($m_p = 1.673\times10^{-27}$ kg) is confined within an atomic nucleus of radius $r \approx 5\times10^{-15}$ m. Taking $\Delta x \approx 2r$, estimate the proton's minimum kinetic energy in MeV.
4. An excited atomic state has a mean lifetime $\tau = 1.0$ ns. Use the energy–time relation to calculate the natural linewidth ΔE in eV and the corresponding frequency spread $\Delta\nu$ in Hz.

D: Quantum Harmonic Oscillator (5 Points)

A particle of mass m in the harmonic potential $V(x) = \frac{1}{2}m\omega^2x^2$ has quantised energy levels

$$E_n = \hbar\omega\left(n+\tfrac{1}{2}\right), \qquad n = 0,1,2,\ldots$$

Model a diatomic molecule as a quantum harmonic oscillator with angular frequency $\omega = 6.55\times10^{13}$ rad s^{-1} and effective reduced mass $\mu = 1.14\times10^{-26}$ kg.

E: Hydrogen Atom Spectral Lines (5 Points)

The energy levels of the hydrogen atom are given by the Bohr formula

$$E_n = -\frac{13.6\ \text{eV}}{n^2}, \qquad n = 1, 2, 3, \ldots$$

1. Determine the ionisation energy of hydrogen (the energy required to remove the electron from the $n = 1$ ground state to $n \to \infty$).
2. An electron transitions from $n = 4$ to $n = 2$. Calculate the energy and wavelength of the emitted photon, and name the spectral series to which this transition belongs.
3. Find the shortest and longest wavelengths in the Lyman series ($n_f = 1$) and identify the region of the electromagnetic spectrum in which they fall.
4. A hydrogen atom in its ground state absorbs a photon of wavelength $\lambda = 97.2$ nm. To which excited state n_f does the electron transition? Show all working and verify using the Bohr formula.

Participation Instructions

- ✓ Write your solutions by hand on sheets of paper or type them on a computer.
- ✓ **Submit your solutions online by Sunday, 15 March 2026, 23:59 UTC+0**
  Website: https://intchc.org/submission
- ✓ You do not need to include the problem statements in your solution document.
- ✓ Show your work to receive full marks.
- ✓ Clearly label each problem and highlight your final answers.
- ✓ You need to score at least **15/17/20 points** as Junior/Youth/Senior to qualify for the Semi-Final Round. See https://intchc.org/age-groups for details.
- ✓ If you have questions, reach out to us at: info@intchc.org

Good Luck!